

Parametric equations 15-12-21

Question 1

[Edexcel C4 Jan 2010 Q7a]

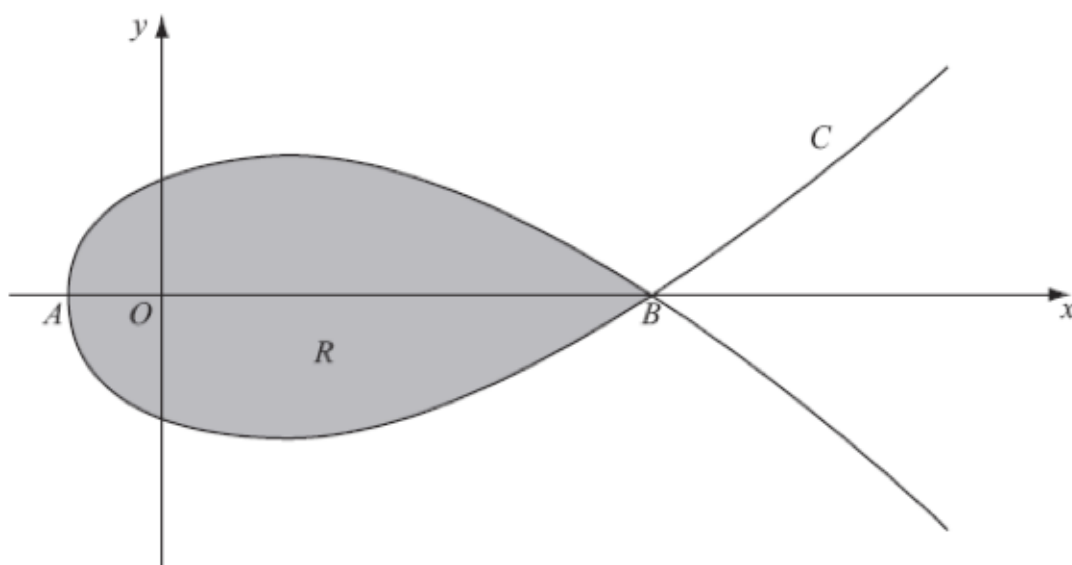


Figure 2

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x -axis at the points A and B .

Find the x -coordinate at the point A and the x -coordinate at the point B .

Correct answer:

or

Their answer:

or

(3 marks)

Question 2

The curve C has parametric equations

$$x = t^2 - 5, \quad y = t^2 - 5, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve C.

Correct answer:

$$y = x^2 - 5$$

Their answer:

$$y = x^2 - 14x + 44$$

Question 3

The curve C has parametric equations

$$x = 3\left(\frac{t}{10}\right)^2 + 1, \quad y = 3\left(\frac{t}{10}\right)^2 + 1, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve C.

Correct answer:

$$y = 3\left(\frac{x}{10}\right)^2 + 1$$

Their answer:

$$y = \frac{3}{10}x + 1$$

Question 6

[Edexcel A2 Specimen Papers P2 Q10a]

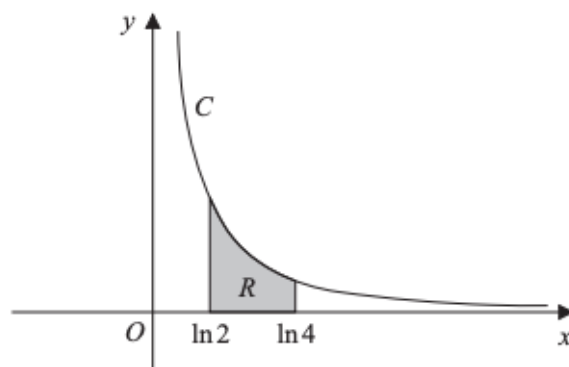


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \ln t, y = \frac{1}{t}, t > \frac{2}{3}$$

State the domain of values of x for the curve C .

Input note: use exact values.

Correct answer:

$$x > \ln \left(\frac{4}{3} \right)$$

Their answer:

$$x > \ln \left(\frac{4}{3} \right)$$

(1 mark)

Question 8

The curve C has parametric equations

$$x = t^2 - 5, \quad y = t^2 - 5, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve C .

Correct answer:

$$y = \left(\frac{x}{2}\right)^2 - 5$$

Their answer:

$$y = \frac{x^2}{4} - 5$$

Question 4

The curve C has parametric equations

$$x = t^2, \quad y = t^2, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve C.

Correct answer:

$$y = x^2$$

Their answer:

$$y = x^2 + 10x + 25$$

Question 5

[Edexcel C4 June 2014 Q5a Edited]

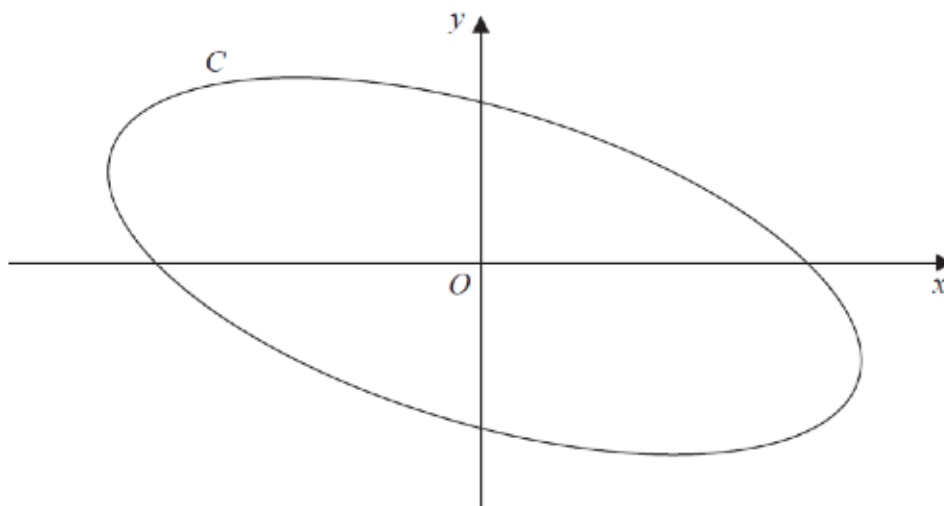


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4 \cos \left(t + \frac{\pi}{6} \right), \quad y = A \sin \left(t + \frac{\pi}{6} \right),$$

Show that

where A is an exact constant to be determined.

Correct answer:

Their answer:

(3 marks)

Question 7

[Edexcel C4 June 2014(R) Q8a]

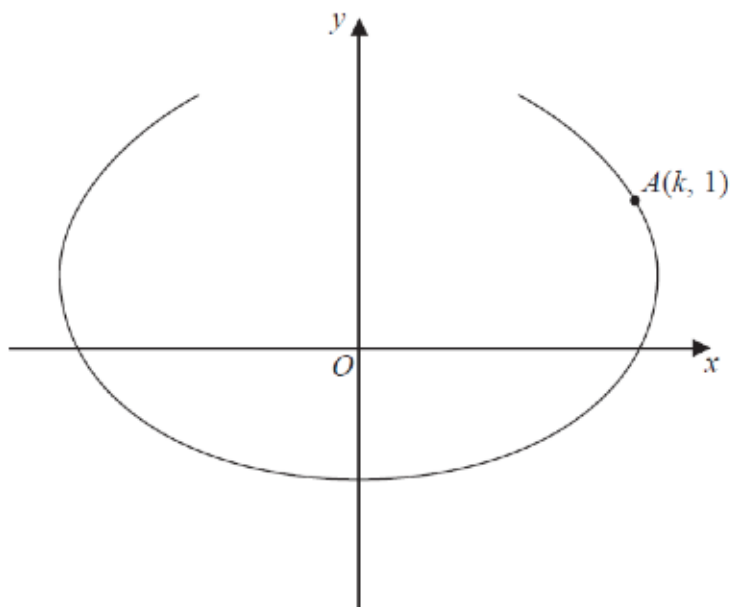


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 4 - \frac{\pi}{2}t, \quad y = 1 - t^2, \quad -3 \leq t \leq 3$$

The point A, with coordinates $(k, 1)$, lies on the curve.

Given that $k > 0$, find the exact value of k .

Correct answer:

$$k = 4 - \frac{\pi}{2}$$

Their answer:

$$4 - \frac{\pi}{2}$$

(2 marks)

Question 9

The curve C has parametric equations

$$x = 4 - \frac{\pi}{2}t, \quad y = 1 - t^2, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve C .

Correct answer:

$$y = 5\left(\frac{x}{2}\right) - 3$$

Their answer:

$$y = \frac{5}{2}x - 3$$

Question 10

The curve C has parametric equations

$$x = \cos^2 t, \quad y = 5 \sin^2 t,$$

Find a Cartesian equation for the curve C .

Correct answer:

$$x + \frac{y}{5} = 1$$

Their answer:

Question 11

[SQA Advanced Higher Maths 2017]

A beam of light passes through the points $B()$ and $T()$.

Obtain parametric equations of the line representing the beam of light.

Correct answer:

''

Their answer:

''

Question 12

The curve C has parametric equations

$$x = 3 - 2t, \quad y = 3 - 2t^2, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve C .

Correct answer:

$$y = 3 - 2x^2$$

Their answer:

Question 13

[Edexcel C4 Jan 2013 Q5d Edited]

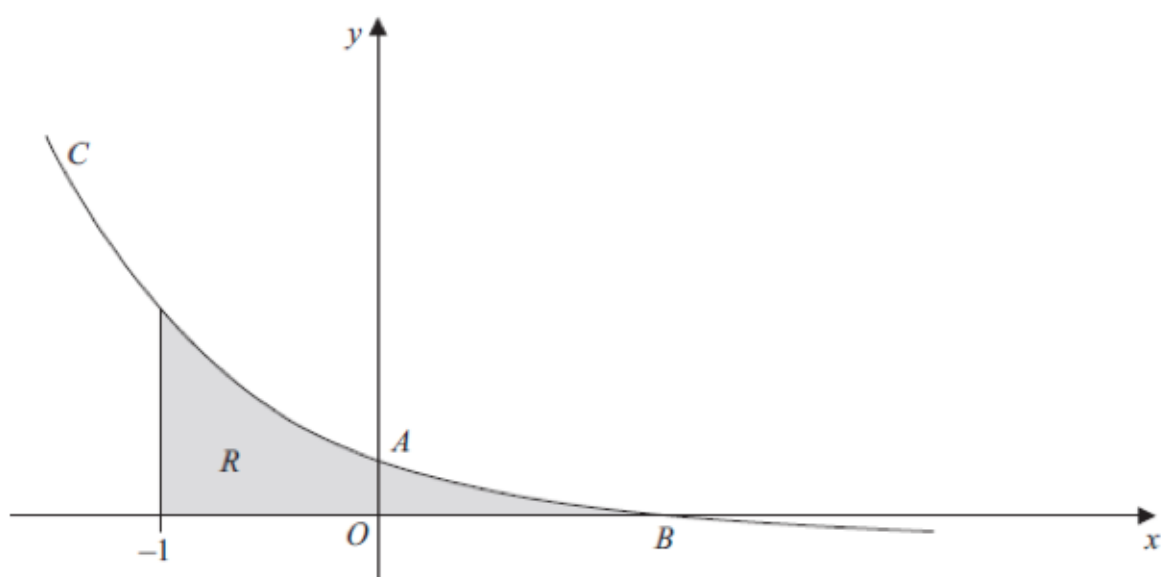


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1.$$

The curve crosses the y -axis at the point A and crosses the x -axis at the point B .

The point A has coordinates $(\quad)$ and the point B has coordinates $(\quad)$.

The region R , as shown shaded in Figure 2, is bounded by the curve C , the line and the x -axis.

Use integration to find the exact area of R .

Correct answer:

$$\frac{15}{2} - 2$$

Their answer:

$$\frac{15}{2 \ln(2)} - 2$$

(6 marks)

Question 14

The curve C has parametric equations

$$x = \frac{1}{7}t^2, \quad y = t^2, \quad t \in \mathbb{R}$$

Find a Cartesian equation for the curve C .

Correct answer:

$$y = \left(\frac{x}{\frac{1}{7}}\right)^2$$

Their answer:

$$y = \frac{x^2}{49}$$

Question 15

[OCR C4 June 2016 Q9i]

A curve has parametric equations $x = 2t^2$, $y = 4t$, for $t \geq 0$.

Find the coordinates of the points where the curve meets the x -axis.

Correct answer:

$(0, 0)$, or $(2, 4)$, or $(8, 8)$

Their answer:

$(0, 0)$, or $(2, 4)$, or $(8, 8)$

(3 marks)

Question 16

The curve C has parametric equations

$x = 6t^2$, $y = 2t$, $t \in \mathbb{R}$

Find a Cartesian equation for the curve C.

Correct answer:

$$y = 2\sqrt{\frac{x}{6}} - 1$$

Their answer:

$$y = 2 - 1$$

Question 17

[Edexcel C4 June 2013 Q4b Edited]

A curve C has parametric equations

$$x = \frac{1}{2}t^2, \quad -\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$$

Find a cartesian equation for C in the form

$$y = f(x),$$

Correct answer:

$$y = \frac{x^2}{2}$$

Their answer:

$$y = \frac{x^2}{2}$$

(3 marks)

Question 18

[Edexcel C4 June 2017 Q1c]

The curve C has parametric equations

$$x = 5 - \frac{6}{t}$$

Show that the cartesian equation for C can be written in the form

$$y = \frac{a}{x+b},$$

where a and b are integers to be determined.

Correct answer:

$$y = \frac{6}{x-5}$$

Their answer:

(3 marks)

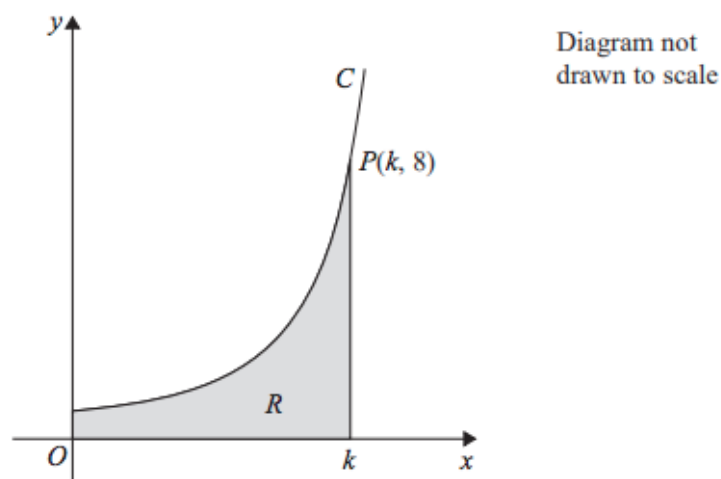
Question 19*[Edexcel C4 June 2017 Q8b Edited]***Figure 4**

Figure 4 shows a sketch of part of the curve C with parametric equations

$$y = \sec^3 \theta \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C , where k is a constant.

It can be shown that

The finite region R , shown shaded in Figure 4, is bounded by the curve C , the y -axis, the x -axis and the line with equation $x = k$.

Show that the area of R can be expressed in the form

where λ , α and β are constants to be determined.

Correct answer:

$$\therefore \beta = \frac{\pi}{3}$$

Their answer:

$$\therefore \beta = \frac{\pi}{3}$$

(4 marks)

Question 20

[Edexcel C4 June 2017 Q8a]

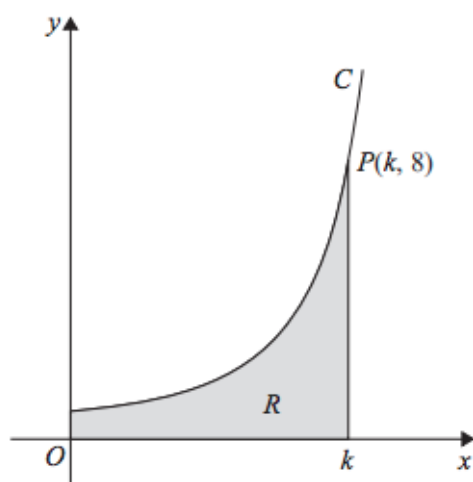


Diagram not
drawn to scale

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$y = \sec^3 \theta \quad 0 \leq \theta < \frac{\pi}{2}$$

The point P lies on C , where k is a constant.

Find the exact value of k .

Correct answer:

Their answer:

(2 marks)

JLL PST - Consolidation 14

Total Marks: 16

Question 17

[Edexcel C3 Jan 2007 Q8iia]

Given that

, and ,

express in terms of y .

Correct answer:

$$\arcsin x = \frac{\pi}{2} - y$$

Their answer:

$$-y + \frac{\pi}{2}$$

(2 marks)

Question 4

[Edexcel C3 Jan 2007 Q2b Edited]

$$f(x) = 1 - 3 + \frac{3}{()^2}$$

It can be shown that $f(x) = \frac{x^2 + x + 1}{()^2}$.

Show that $x^2 + x + 1 > 0$ for all values of x .

Input note: write $x^2 + x + 1$ in a form that explicitly show it is positive for all x .

Correct answer:

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

Their answer:

$$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

(3 marks)

Question 11

[Edexcel C3 Jan 2007 Q6a Edited]

The function f is defined by

$$f: x \rightarrow \ln(), \quad \text{and } x \in$$

Find the inverse function of f .

Correct answer:

$$f^{-1}(x) = 2 - \frac{1}{2}e^x$$

Their answer:

$$2 - \frac{1}{2}e^x$$

(4 marks)

Question 3

[Edexcel C3 Jan 2007 Q2a Edited]

$$f(x) = 1 - 3 + \frac{3}{()^2}$$

Show that $f(x) = \frac{x^2 + ax + b}{()^2}$, where a and b are constants to be found.

Correct answer:

,

Their answer:

,

(4 marks)

Question 16

[Edexcel C3 Jan 2007 Q8i Edited]

Prove that

$$\sec^2 x - \csc^2 x = a \tan^2 x + b \cot^2 x$$

where a and b are constants to be found.

Correct answer:

,

Their answer:

,

(3 marks)

JLL PST - Consolidation 14

Total Marks: 47

Question 18

[Edexcel C3 Jan 2007 Q8iib Edited]

Given that

, and ,

It can be shown that $\arcsin x = \frac{\pi}{2} - y$

Hence evaluate . Give your answer in terms of π .

Correct answer:

$$\arccos x + \arcsin x = \frac{\pi}{2}$$

Their answer:

$$\frac{\pi}{2}$$

(1 mark)

Question 1

[Edexcel C3 Jan 2007 Q1a Edited]

By writing as $\sin()$, show that

$$\sin 3\theta = a \sin \theta - b \sin^3 \theta$$

where a and b are constants to be found.

Correct answer:

,

Their answer:

,

(5 marks)**Question 2***[Edexcel C3 Jan 2007 Q1b Edited]*

It can be shown that

$$\sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

Given that $\sin \theta = \frac{1}{2}$, find the exact value of $\sin 3\theta$.**Correct answer:****Their answer:****(2 marks)****Question 5***[Edexcel C3 Jan 2007 Q3b Edited]*The curve C has equation $y = x^3 - 3x^2 + 2x$.It can be shown that the point P lies on C .Find the value of x at P .**Correct answer:****Their answer:**

(4 marks)**Question 6***[Edexcel C3 Jan 2007 Q3c Edited]*

The curve C has equation .

It can be shown that the point lies on C .

It is given that at P .

Find an equation of the normal to C at P . Give your answer in the form , where m and c are exact constants.

Correct answer:**Their answer:****(4 marks)****Question 7***[Edexcel C3 Jan 2007 Q4i]*

The curve C has equation $y = \frac{x}{9+x^2}$.

Use calculus to find the coordinates of the turning points of C .

Correct answer:

$$, y = \frac{1}{6} \text{ or } , y = -\frac{1}{6}$$

Their answer:

$$, y = \frac{1}{6} \text{ or } y = -\frac{1}{6}$$

(6 marks)

Question 8

[Edexcel C3 Jan 2007 Q4ii]

Given that $y = (1 + e)^{\frac{3}{2}}$, find the value of y at $x = \frac{1}{2} \ln 3$.

Correct answer:

$$y = 18$$

Their answer:

$$18$$

(5 marks)

Question 9

[Edexcel C3 Jan 2007 Q5a]

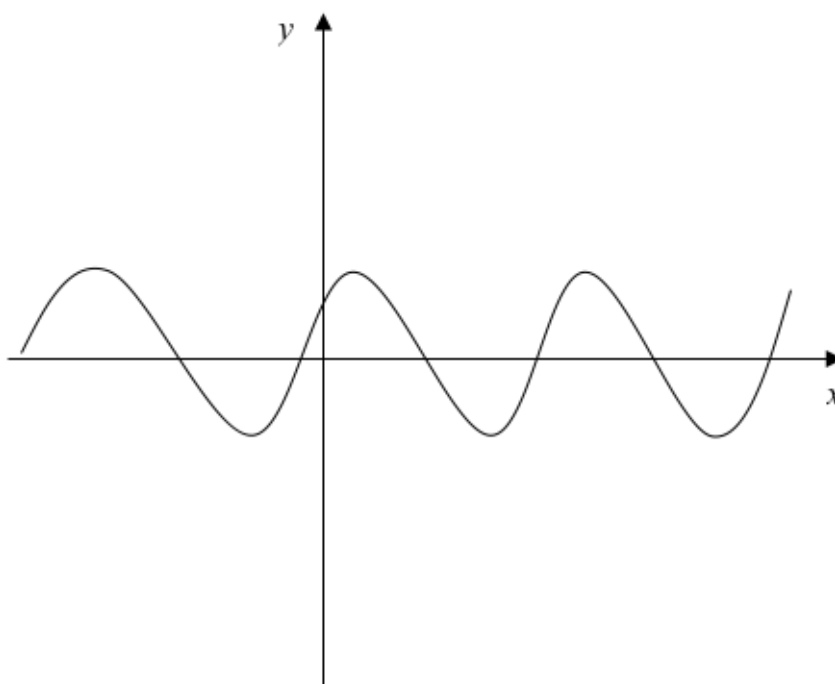
Figure 1

Figure 1 shows an oscilloscope screen.

The curve on the screen satisfies the equation .

Express the equation of the curve in the form $y = R \sin()$, where R and α are constants, and

$$0 < \alpha < \frac{\pi}{2}.$$

Input note: give your answers as exact values.

Correct answer:

$$, \alpha = \frac{\pi}{3}$$

Their answer:

$$, \alpha = \frac{\pi}{3}$$

(4 marks)

Question 10

[Edexcel C3 Jan 2007 Q5b Edited]

Figure 1

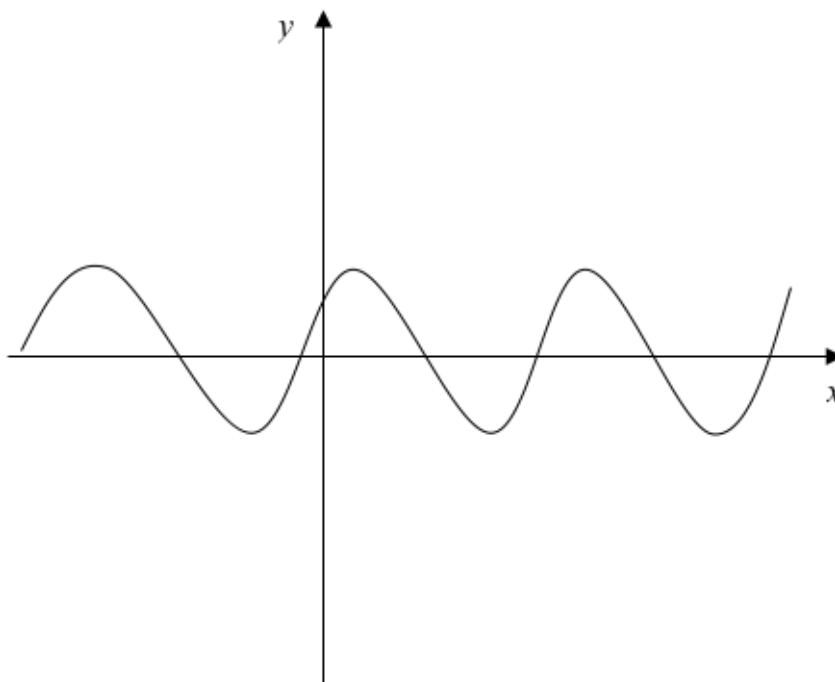


Figure 1 shows an oscilloscope screen.

The curve on the screen satisfies the equation .

It can be shown that $y = 2 \sin \left(x + \frac{\pi}{3} \right)$

Find the values of x , for which .

Input note: give your solutions as exact values.

Correct answer:

$$x = \frac{\pi}{2} \text{ or } x = 6$$

Their answer:

$$x = \frac{\pi}{2} \text{ or } x = 6$$

(4 marks)

Question 11

[Edexcel C3 Jan 2007 Q6a Edited]

The function f is defined by

$$f: x \rightarrow \ln(x), \quad \text{and } x \in$$

Find the inverse function of f .

(4 marks)

Question 12

[Edexcel C3 Jan 2007 Q6b Edited]

The function f is defined by

$$f: x \rightarrow \ln(x), \quad \text{and } x \in$$

$$\text{It can be shown that } f^{-1}(x) = 2 - \frac{1}{2}e^x$$

Write down the range of f^{-1} .

Correct answer:

$$f^{-1}(x) < 2$$

Their answer:

$$f^{-1}(x) < 2$$

(1 mark)

Question 13

[Edexcel C3 Jan 2007 Q6d Edited]

The function f is defined by

$$f: x \rightarrow \ln(x), \quad \text{and } x \in$$

It can be shown that $f^{-1}(x) = 2 - \frac{1}{2}e^x$

The graph of f crosses the graph of $y = f^{-1}(x)$ at .

The iterative formula

$$x = -\frac{1}{2}e^{x_n}, \quad x_0 = -0.3$$

is used to find an approximate value for k .

Calculate the values of x_1 and x_2 , giving your answer to 4 decimal places.

Correct answer:

$$x_1 = -0.3704 \text{ and } x_2 = -0.3452$$

Their answer:

$$x_1 = -0.3704 \text{ and } x_2 = -0.3452$$

(2 marks)

Question 15

[Edexcel C3 Jan 2007 Q7c]

$$f(x) = x^4 - 4x - 8$$

Given that $f(x) = (x^3 + ax^2 + bx + c)$, find the values of the constants a , b and c .

Correct answer:

''

Their answer:

''

(3 marks)

Question 17

[Edexcel C3 Jan 2007 Q8iia]

Given that

, and ,

express in terms of y .

(2 marks)

Year 13 Consolidation 11

Total Marks: 9

Question 5

[Edexcel C3 Jan 2006 Q4b]

Given that $x = 4 \sin \theta$, find $\frac{dx}{d\theta}$ in terms of x .

Correct answer:

$$\frac{dx}{d\theta} = 4 \cos \theta = 4 \sqrt{1 - \sin^2 \theta} = 4 \sqrt{1 - \left(\frac{x}{4}\right)^2}$$

Their answer:

(5 marks)

Question 15

[Edexcel C3 Jan 2006 Q8d Edited]

The functions f and g are defined by

$$f: x \mapsto e^x, \quad x \in \mathbb{R}$$

$$g: x \mapsto e^x, \quad x \in \mathbb{R}$$

It can be shown that $gf(x) = 4e^x$

Find the value of x for which $d(gf(x)) = 3$, giving your answer to 3 significant figures.

Correct answer:

$$-0.418$$

Their answer:

-0.418

(4 marks)

Year 13 Consolidation 11

Total Marks: 55

Question 2

[Edexcel C3 Jan 2006 Q3]

The point P lies on the curve with equation $y = \ln\left(\frac{1}{3}x\right)$. The x -coordinate of P is 3.

Find an equation of the normal to the curve at the point P in the form $ax + by = c$, where a and b are constants.

Correct answer:

Their answer:

(5 marks)

Question 3

[Edexcel C3 Jan 2006 Q4ai]

Differentiate with respect to x

$$x^2e$$

Correct answer:

$$d(x^2e) = 3x^2e + 2xe$$

Their answer:

$$2xe + 3x^2e$$

(4 marks)

Question 4

[Edexcel C3 Jan 2006 Q4a]ii]

Differentiate with respect to x

$$\frac{\cos(2x^3)}{9x^2}$$

Correct answer:

$$\frac{d(\cos(2x^3))}{dx} = \frac{-18x^3 \sin(2x^3) - 3 \cos(2x^3)}{9x^2}$$

Their answer:

$$\frac{-6x^3 \sin(2x^3) - \cos(2x^3)}{3x^2}$$

(4 marks)

Question 5

[Edexcel C3 Jan 2006 Q4b]

Given that $x = 4 \sin \theta$, find $\frac{dx}{d\theta}$ in terms of x .

(5 marks)

Question 6

[Edexcel C3 Jan 2006 Q6a Edited]

$$f(x) = 12 \cos x - 4 \sin x$$

Given that $f(x) = R \cos(x - \alpha)$, where $0 \leq \alpha < 90^\circ$, find the value of R and the value of α .

Input note: give both values correct to 1 decimal place.

Correct answer:

12.6 and 18.4 °

Their answer:

12.6 and 18.4 °

(4 marks)

Question 7

[Edexcel C3 Jan 2006 Q6b Edited]

$$f(x) = 12 \cos x - 4 \sin x$$

It can be shown that

Hence solve the equation

for $0 \leq x < 360^\circ$, giving your answers to one decimal place.

Correct answer:

38.0 ° and 285.2 °

Their answer:

38.0 ° and 285.2 °

(5 marks)

Question 8

[Edexcel C3 Jan 2006 Q6ci Edited]

$$f(x) = 12 \cos x - 4 \sin x$$

It can be shown that

Write down the minimum value of .

Correct answer:

Their answer:

(1 mark)

Question 9

[Edexcel C3 Jan 2006 Q6cii Edited]

$$f(x) = 12 \cos x - 4 \sin x$$

It can be shown that

Find, to 2 decimal places, the smallest positive value of x for which the minimum value of $f(x)$ occurs.

Correct answer:

$$161.57^\circ$$

Their answer:

$$161.57^\circ$$

(2 marks)

Question 10

[Edexcel C3 Jan 2006 Q7ai Edited]

Express

.

where $x \neq (n - \frac{1}{4})\pi$, $n \in \mathbb{Z}$

in terms of $\sin$ and $\cos$ only.

Correct answer:

Their answer:

(2 marks)

Question 11

[Edexcel C3 Jan 2006 Q7a] Edited]

Show that

$$\frac{1}{2} \cos^2 x \equiv \cos^2 x - \cos x \sin x - a$$

where a is a constant to be found.

Correct answer:

$$a = \frac{1}{2}$$

Their answer:

0.5

(3 marks)

Question 12

[Edexcel C3 Jan 2006 Q7c]

Solve, for x ,

giving your answers in terms of π .

Correct answer:

$$\theta = \frac{\pi}{8} \text{ or } \theta = 8 \text{ or } \theta = 8 \text{ or } \theta = 8$$

Their answer:

$$\theta = \frac{\pi}{8} \text{ or } \theta = 8 \text{ or } \theta = 8 \text{ or } \theta = 8$$

(4 marks)

Question 13

[Edexcel C3 Jan 2006 Q8a Edited]

The functions f and g are defined by

$$f: x \mapsto 4x, \quad x \in \mathbb{R}$$

$$g: x \mapsto e, \quad x \in \mathbb{R}$$

Find the composite function gf in the form $gf(x) = ae$

Correct answer:

$$gf(x) = 4e$$

Their answer:

$$4e$$

(4 marks)

Question 14

[Edexcel C3 Jan 2006 Q8c Edited]

The functions f and g are defined by

$$f(x) = 4x, \quad x \in \mathbb{R}$$

$$g(x) = x + e, \quad x \in \mathbb{R}$$

It can be shown that $gf(x) = 4e$

Write down the range of f .

Correct answer:

$$gf(x) > 0$$

Their answer:

$$gf(x) > 0$$

(1 mark)

Question 15

[Edexcel C3 Jan 2006 Q8d Edited]

The functions f and g are defined by

$$f(x) = 4x, \quad x \in \mathbb{R}$$

$$g(x) = x + e, \quad x \in \mathbb{R}$$

It can be shown that $gf(x) = 4e$

Find the value of x for which $d(gf(x)) = 3$, giving your answer to 3 significant figures.

(4 marks)

Question 1

[Edexcel C3 Jan 2006 Q2]

Express

$$\frac{2x^2+3x}{(x-1)(x+2)} - \frac{6}{x^2-x-2}$$

as a single fraction in its simplest form.

Correct answer:

.

Their answer:

.

(7 marks)
